PROPAGATION OF SMALL PERTURBATIONS IN GASES CONTAINING SUSPENDED SOLIDS

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Inzhenerno-Fizicheskii Zhurnal, Vol. 14, No. 4, pp. 716–721, 1968

UDC 534.21:532.529.5

The acoustical properties of a gas-solids suspension are investigated. A combined (mechanical, thermodynamic) approach to the study of the propagation of small perturbations in mixtures is adopted.

The theory of sound absorption in disperse systems, due to S. M. Rytov, V. V. Vladimirskii, and M. D. Galanin [1], assumes that the medium is barotropic.

In [2] P. P. Zolotarev introduced temperature effects and, taking into consideration the compressibility of the suspended component, obtained very clumsy formulas for the dispersion of the speed and absorption of sound at low frequencies.

The present paper analyzes the effects accompanying the propagation of acoustical disturbances in a monodisperse gas-particle system. Simplified relations for the speed of sound and the attenuation constant are then derived.

§1. When small low-frequency perturbations are generated in a gas-solids suspension, the disturbed mixture will be essentially in equilibrium. In the limiting case of small frequencies the suspended particles can follow the acoustic wave with respect to both displacement and temperature variation. "Macroscopically," the propagation process will be adiabatic, but at small frequencies the temperature differences between the particles and the gas will be able to adjust themselves, and "microscopically" the process will be isothermal [3]. Knowing the mechanical and thermodynamic constants of the components, one can easily calculate the values of the compressibility and the speed of sound in the mixture corresponding to this case.

For the adiabatic compressibility we have the following relation [4]:

$$\frac{\beta}{\gamma}=\beta-\frac{T_0\alpha_v^2}{\rho_0 c_p}. \tag{1}$$

Hence, by virtue of the additivity of the quantities α_V, β, and ρc_p, the adiabatic-isothermal value of the compressibility can be represented in the form*

$$\left(\frac{\overline{\beta}}{\gamma}\right)_0=(1-\varepsilon_0)\beta-\frac{T_0[(1-\varepsilon_0)\alpha_v]^2}{(1-\varepsilon_0)\rho_0 c_p+\varepsilon_0\rho_m c_m}$$

or, using the known thermodynamic relation $c_p - c_V = T_0\alpha_V^2/\rho_0\beta$,

$$\left(\frac{\overline{\beta}}{\gamma}\right)_0=\frac{(1-\varepsilon_0)\beta}{\overline{\gamma}_0}, \tag{2}$$

where

$$\overline{\gamma}_0=\frac{c_p+\dfrac{\varepsilon_0}{1-\varepsilon_0}\,\delta c_m}{c_v+\dfrac{\varepsilon_0}{1-\varepsilon_0}\,\delta c_m}.$$

The quantity $\overline{\gamma}_0$ may be called the equilibrium adiabatic exponent.

From (2) we obtain an expression for the equilibrium speed of sound:

$$\overline{c}_0=\left[\frac{(1-\varepsilon_0)\beta\overline{\rho}_0}{\overline{\gamma}_0}\right]^{-\frac{1}{2}}. \tag{3}$$

As the frequency of the perturbations increases, the particles are less and less able to follow the resulting motions and temperature changes of the gas. In the high-frequency limit the gas temperature fluctuations take place so rapidly that any heat transfer between the components is excluded. The compressions and expansions in the mixture will be adiabatic even on the "microscopic" level. However, the effective density of the mixture approaches the density of the gas. Its value was found in [2] and is represented by the following relation:

$$\frac{1}{\overline{\rho}_\infty}=\frac{\varepsilon_0}{\rho_m}+\frac{1-\varepsilon_0}{\rho_0}.$$

Now, the adiabatic-adiabatic compressibility of the mixture is determined from (1) in the form

$$\left(\frac{\overline{\beta}}{\gamma}\right)_\infty=\frac{(1-\varepsilon_0)\beta}{\gamma}, \tag{4}$$

and for the phase velocity of propagation of the acoustic wave we have

$$\overline{c}_\infty=\left[\frac{(1-\varepsilon_0)\beta\overline{\rho}_\infty}{\gamma}\right]^{-\frac{1}{2}}. \tag{5}$$

The quantity $\overline{c}_\infty$ is usually called the "frozen" speed of sound. In both limiting cases the speed of sound and hence the wave number $k = \omega/c$ are real, i.e., there are no absorptions and phase shifts.

*We note that in [3] some of the definitions are inaccurate. The correct formula for the adiabatic-isothermal compressibility of the emulsion (p. 908) has the form

$$\varepsilon\beta_1+(1-\varepsilon)\beta_2-\Theta\,\frac{[\varepsilon\alpha_1+(1-\varepsilon)\alpha_2]^2}{\varepsilon\rho_1 c_{p1}+(1-\varepsilon)\rho_2 c_{p2}},$$

and

$$c_1=[\varepsilon\rho_1+(1-\varepsilon)\rho_2]^{-\frac{1}{2}}\times$$
$$\times\left\{\varepsilon\beta_1+(1-\varepsilon)\beta_2-\Theta\,\frac{[\varepsilon\alpha_1+(1-\varepsilon)\alpha_2]^2}{\varepsilon\rho_1 c_{p1}+(1-\varepsilon)\rho_2 c_{p2}}\right\}^{-\frac{1}{2}}.$$

In the intermediate frequency region friction, which tends to equalize the particle and gas velocities, and the periodic irreversible heat transfer from the gas to the particles and back lead to significant energy dissipation. As a result of the relative slowness of the entrainment of the particles by the gas and the equalization of their temperatures dispersion of the sound occurs comparatively early.

§2. We will examine the behavior of the dispersion curve between the asymptotic values obtained for the velocity.

Assuming that viscosity and thermal conductivity effects are important only in the gas-particle interaction processes, we can write the system of acoustical equations of conservation of mass, momentum, and energy in the following form [2, 1]:

$$\frac{\partial}{\partial t}\left[(1-\varepsilon_0)\rho-\rho_0\varepsilon\right]+(1-\varepsilon_0)\rho_0\frac{\partial u}{\partial x}=0, \qquad (6)$$

$$\frac{\partial\varepsilon}{\partial t}+\varepsilon_0\frac{\partial w}{\partial x}=0, \qquad (7)$$

$$\frac{\partial}{\partial t}\left[(1-\varepsilon_0)\rho_0 u+\varepsilon_0\rho_m w\right]=-\frac{\partial p}{\partial x}, \qquad (8)$$

$$\frac{\partial}{\partial t}(\rho_m w-\rho_0 u)=\frac{6\pi r\mu}{V}(u-w), \qquad (9)$$

$$\frac{\partial}{\partial t}\left[(1-\varepsilon_0)\rho_0 c_p T+\varepsilon_0\rho_m c_m\Theta\right]=(1-\varepsilon_0)\alpha_v T_0\frac{\partial p}{\partial t}, \qquad (10)$$

$$\frac{\partial\Theta}{\partial t}=\frac{3\alpha}{\rho_m c_m r}(T-\Theta). \qquad (11)$$

In Eq. (9) we have taken the Stokes value of the particle drag coefficient, which is valid only for waves that do not exhibit very high frequency. However, in the case of a Stokes approximation of the law of particle entrainment the Nusselt number can be taken equal to two. Then for the heat transfer coefficient in heat transfer equation (11) we have $\alpha = \lambda/r$.

To close system (6)–(11) we add the acoustical equation of state for the gas:

$$\frac{\rho}{\rho_0}=\beta p-\alpha_v T. \qquad (12)$$

We will find the solution of system (6)–(12) in the form of a harmonic plane wave, writing all the dependent variables in the form

$$f=f^*\exp\left[-i(\omega t-kx)\right]. \qquad (13)$$

As a result we obtain the following dispersion relation:

$$\frac{k^2}{\omega^2}=\frac{(1-\varepsilon_0)\beta\rho_0}{\bar\gamma_{\text{eff}}}\,\frac{(1-\varepsilon_0)\rho_0+\varepsilon_0\rho_m B_\mu}{(1-\varepsilon_0)\rho_0+\varepsilon_0\rho_0 B_\mu}. \qquad (14)$$

Here,

$$\bar\gamma_{\text{eff}}=\frac{c_p+\frac{\varepsilon_0}{1-\varepsilon_0}\delta c_m B_\lambda}{c_v+\frac{\varepsilon_0}{1-\varepsilon_0}\delta c_m B_\lambda}, \qquad (15)$$

$$B_\mu=\frac{w}{u}=\frac{1+i\delta^{-1}\tau_\mu\omega}{1+i\tau_\mu\omega}=\frac{1+i\frac{4}{9}\xi_\mu}{1+i\frac{4}{9}\delta\xi_\mu}, \qquad (16)$$

$$B_\lambda=\frac{\Theta}{T}=\frac{1}{1+i\tau_\lambda\omega}=\frac{1}{1+\frac{2}{3}\delta\frac{c_m}{c_p}\xi_\lambda}. \qquad (17)$$

For most gases and metal particles the term $1/3(\mu c_m/\lambda)$ is equal to 2/9, and hence the time characteristics of thermal and dynamic particle relaxation are effectively equal.

As $\xi_\mu \to 0$ and $\xi_\lambda \to 0$, B_μ and B_λ tend to unity and (14) gives the expression for the equilibrium velocity of sound (3).

Exactly as in [1, 2], it can be shown that our final formulas (14)–(17) are valid up to $\omega\tau_\mu = 1$ and $\omega\tau_\lambda = 1$. However, it is easy to arrive at the conclusion that the latter relations should also give the correct limiting value of the phase speed of sound as $\xi_\mu \to \infty$ and $\xi_\lambda \to \infty$ (or $\omega\tau_\mu \to \infty$, $\omega\tau_\lambda \to \infty$).

In the limit from (14) we obtain the frozen speed of sound (5).

The frequency at which the period of the generated wave is equal to the particle relaxation time is the critical frequency. Transition through this frequency corresponds to transition from the equilibrium to the frozen speed of sound.

In the case of low dispersion, from (14) for the complex wave number we have approximately

$$k=\frac{\omega}{c_0}\left[1+\frac{1}{2}\times\frac{\frac{\varepsilon_0}{1-\varepsilon_0}(\gamma-1)\delta c_m c_v}{\left(c_p+\frac{\varepsilon_0}{1-\varepsilon_0}\delta c_m\right)\left(c_v+\frac{\varepsilon_0}{1-\varepsilon_0}\delta c_m\right)}(B_\lambda-1)+\frac{1}{2}\frac{\varepsilon_0(1-\varepsilon_0)(\delta-1)}{1+\varepsilon_0(\delta-1)}(B_\mu-1)\right]. \qquad (18)$$

Keeping the real and imaginary parts separate, we obtain the law of dispersion of velocity and the attenuation constant:

$$\frac{1}{c}=\frac{1}{c_0}\times\left[1-\frac{2}{9}\frac{\frac{\varepsilon_0}{1-\varepsilon_0}(\gamma-1)\delta^3\left(\frac{c_m}{c_p}\right)^2 c_m c_v}{\left(c_p+\frac{\varepsilon_0}{1-\varepsilon_0}\delta c_m\right)\left(c_v+\frac{\varepsilon_0}{1-\varepsilon_0}\delta c_m\right)}\times\frac{\xi_\lambda^2}{1+\frac{4}{9}\delta^2\left(\frac{c_m}{c_p}\right)^2\xi_\lambda^2}-\frac{1}{2}\frac{\varepsilon_0(1-\varepsilon_0)(\delta-1)^2\delta}{1+\varepsilon_0(\delta-1)}\frac{\left(\frac{4}{9}\xi_\mu\right)^2}{1+\left(\frac{4}{9}\delta\right)^2\xi_\mu^2}\right], \qquad (19)$$

$$\delta^* =$$

$$= \frac{\omega}{\bar{c}_0}\left[\frac{1}{3}\,\frac{\frac{\varepsilon_0}{1-\varepsilon_0}(\gamma-1)\gamma^{-1}\delta^2 c_m^2}{\left(c_p+\frac{\varepsilon_0}{1-\varepsilon_0}\delta c_m\right)\left(c_v+\frac{\varepsilon_0}{1-\varepsilon_0}\delta c_m\right)}\times\right.$$

$$\times\frac{\xi_\lambda}{1+\frac{4}{9}\delta^2\left(\frac{c_m}{c_p}\right)^2\xi_\lambda^2}+$$

$$\left.+\frac{2}{9}\,\frac{\varepsilon_0(1-\varepsilon_0)(\delta-1)^2}{1+\varepsilon_0(\delta-1)}\,\frac{\xi_\mu}{1+\left(\frac{4}{9}\delta\right)^2\xi_\mu^2}\right]. \qquad (20)$$

For sufficiently rarefied systems, when it is possible to neglect not only the volume but also the mass particle concentration as compared with unity, (20) reduces to the form

$$\delta^* =$$

$$= \frac{\omega}{c}\left[\frac{1}{3}\,\varepsilon_0(\gamma-1)\delta^2\left(\frac{c_m}{c_p}\right)^2\frac{\xi_\lambda}{1+\frac{4}{9}\delta^2\left(\frac{c_m}{c_p}\right)^2\xi_\lambda^2}+\right.$$

$$\left.+\frac{2}{9}\,\varepsilon_0(\delta-1)^2\,\frac{\xi_\mu}{1+\left(\frac{4}{9}\delta\right)^2\xi_\mu^2}\right]. \qquad (21)$$

And, finally, in the case $\xi_\mu \ll 1$ we have the following relation for the relative sound attenuation:

$$\frac{cr^2\delta^*}{\varepsilon_0\nu} =$$

$$= \left[\frac{4}{9}(\delta-1)^2+\frac{2}{3}(\gamma-1)\delta^2\left(\frac{c_m}{c_p}\right)^2\mathrm{Pr}\right]\xi_\mu^2. \qquad (22)$$

The results of the theory are in satisfactory agreement with the experimental data of [5].

NOTATION

$a = \lambda/\rho_0 c_p$ is the thermal diffusivity of the gas; c is the speed of sound in the gas; $\bar{c}_0$ is the equilibrium speed of sound; $\bar{c}_\infty$ is the frozen speed of sound; c_m is the specific heat of the particle material; c_v and c_p are the specific heats of the gas at constant volume and pressure; f^* is the amplitude; k is the wave number; p is the sound pressure; r is the particle radius; t is the time; T is the perturbation of gas temperature; u is the gas velocity; V is the particle volume; w is the particle velocity; α is the heat transfer coefficient; α_v is the coefficient of volume expansion of the gas; β is the isothermal gas compressibility; $\gamma = c_p/c_v$ is the ratio of specific heats; γ_{eff} is the effective adiabatic exponent; $\delta = \rho_m/\rho_0$ is the density ratio; ε is the perturbation of particle volume concentration; Θ is the deviation of particle temperature from its value in the undisturbed state; λ is the thermal conductivity; μ, ν denote the dynamic and kinematic viscosities of gas; ρ is the deviation of gas density from its value in the undisturbed medium; ρ_m is the density of the particle material; $\bar{\rho}_0 = \varepsilon_0\rho_m + (1-\varepsilon_0)\rho_0$; $\tau_\lambda = 1/3(c_m\rho_m r^2/\lambda)$ and $\tau_\mu = 2/9(\rho_m r^2/\nu)$ are the thermal and dynamic particle relaxation times; $\xi_\lambda = \omega r^2/2a$ and $\xi_\mu = \omega r^2/2\nu$ are parameters; ω is the cyclic frequency; $B_\lambda = \Theta/T$; $B_\mu = w/u$ is the velocity ratio; $Nu = 2\alpha r/\lambda$ is the Nusselt number; $Pr = \nu/a$ is the Prandtl number; $(\bar{\ })$ is the mean over the volume of the mixture; $(\ )_0$ is the undisturbed value; low-frequency value; $(\ )_\infty$ is the high-frequency value.

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11 July 1967

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